Lecture 1 Predicates and universe of discourse

Def'n A *predicate* is an expression

$$P(x_1, \dots, x_n)$$

in the variables $x_1, \dots, x_n$ that becomes a proposition when the variables are replaced by specific objects.

Def'n The *truth set* of a predicate $P(x_1, \dots, x_n)$ is the set of values in Ω of variables $x_1, \dots, x_n$ for which the predicate becomes a true proposition, where Ω is a given *universe of discourse*.

Example We have the predicate $P(x)$ given by

$$x^2 - 2 = 0. \qquad \star$$

If $\Omega = \mathbb{Q}$ then the truth set of $P(x)$ is the empty set $\emptyset$. Indeed, if $x \in \mathbb{Q}$ s.t. $(\star)$, then there exist $a, b \in \mathbb{Z}_{>0}$ s.t.

(i) $(a, b) = 1$

(ii) $a^2 = 2b^2$

By (ii) in particular we have $2 \mid a^2$. But we have the prime factorisation

$$a^2 = p_1^{2e_1} \cdots p_g^{2e_g}$$

where

$$a = p_1^{e_1} \cdots p_g^{e_g} \qquad (p_1 < \cdots < p_g)$$

is the prime factorisation of a. Thus $p_1 = 2$. Hence

$$2^2 \mid 2b^2$$

$$2 \mid b^2$$

But as above, we have the prime factorisation

$$b^2 = q_1^{2e_1'} \cdots q_h^{2e_h'},$$

$$b = q_1^{e_1'} \cdots q_h^{e_h'} \qquad (q_1 < \cdots < q_h)$$

is the prime factorisation of b. Thus $q_1 = 2$, which contradicts (i). Therefore it follows that the truth set of $P(x)$ is the empty set $\emptyset$.

The above argument uses the Fundamental Theorem of Arithmetic.