Lecture 5 Posets

A *partially ordered set* is an ordered pair $(S, \leqslant)$, where S is a set and $\leqslant \subseteq S \times S$ is such that

R. $\forall x \in S : x \leqslant x$

A. $\forall x, y \in S : x \leqslant y \;\&\; y \leqslant x \Rightarrow x = y$

T. $\forall x, y, z \in S : x \leqslant y \;\&\; y \leqslant z \Rightarrow x \leqslant z$

and $\forall x, y \in S$ we write $x \leqslant y$ precisely when $(x, y) \in \leqslant$.

Example Let $S = P(X)$, where X is any given set and let $x \leq y$ precisely when $x \subseteq y$. Here as before $P(X)$ denotes the power set of X.